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Dynamic Assessment of Tunnels Subjected to Moving Train Loads (Case Study: Tunnel of Tehran Subway Line 3)

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ABSTRACT

The expansion of urban populations necessitates improvements in construction and the development of urban infrastructure. The creation of subway tunnels in urban centers has consistently faced considerable challenges and constraints. This study specifically focuses on one such issue: the intersection of subway lines with existing railway lines. The main aim of this research is to model the dynamic load imposed by a moving train on a section of the Tehran Metro Line 3 tunnel as it passes beneath the Tehran-Ahvaz intercity railway. To accomplish this, the dynamic load generated by the train's movement is simulated, and a corresponding function for this load is derived. This function is subsequently incorporated into a numerical model of the tunnel, which is developed using the Finite Difference Method (FDM). Given that the tunnel is situated within soil, the environment surrounding it is regarded as a continuous equivalent medium. The results of this modeling are then compared with two analytical approaches. The comparison demonstrates a strong correlation between the results, thereby validating the accuracy of the findings from the computational model. Additionally, by assessing the most critical operational conditions for trains on the Tehran-Ahvaz railway, a safety factor of 2.34 is established, indicating the design's robustness under the specified conditions.

Keywords: Dynamic Loading; Train Dynamic Load; Numerical Modeling; Dynamic Analysis of Tunnels; Finite Difference Method

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1. Introduction

The stability of tunnel structures is significantly influenced by variations in train speeds and loads, leading to dynamic responses that can compromise structural integrity. A thorough comprehension of these factors is crucial for ensuring the safety and longevity of tunnel systems. The following sections outline the key impacts of train dynamics on tunnel stability and examine potential mitigation strategies.

Increased train speeds lead to heightened peak particle velocities within tunnels, especially at the arch, which may exceed the responses of the adjacent rock mass [1]. Extended cyclic loading from train operations can cause shear slippage in rock formations, exacerbating tunnel deformation, particularly in jointed and stratified geological environments [2].

Inadequacies in tunnel linings can amplify vibrational effects, with the characteristics of the train and its axle load being critical factors influencing dynamic responses.

Mitigation strategies identified in the literature include: the implementation of tunnels with variable cross-sectional profiles to reduce adverse pressure effects and enhance aerodynamic efficiency, thereby bolstering overall stability and conducting regular assessments of tunnel linings and surrounding geological conditions to enable early detection and correction of stiffness degradation and structural defects before they develop into significant issues [3]. While these strategies can mitigate certain risks, the inherent challenges posed by geological variability and the dynamic nature of train operations highlight the need for ongoing research and adaptive engineering practices to maintain tunnel safety and functionality [4].

One of the limitations brought about by the design of Tehran metro line 3 is the passing of this line under the trans-country railroad. In the initial design, this part of the tunnel was constructed using the cut and cover method. In fact, after opening, precast concrete beams are placed on the ceiling of the metro tunnel like a bridge, which the trans-country railroad will cross over. Therefore, examination of the effect of the dynamic load caused by the movement of railroad trains over this bridge plays a major role in the stability of the tunnel. Usually encountering dynamic problems is more complicated and takes longer than solving static problems. For this reason, it is often attempted in usual and frequent jobs, with appropriate estimations, to replace dynamic loads with equivalent static loads and instead increase their effects with an incremental coefficient named the coefficient of impact. As such, in designing some projects with relatively less importance, engineers can reach acceptable results by simpler methods. This method is still used as a common model in the calculations related to the effect of dynamic loads on bridges. What is obvious is that obtaining appropriate relations for the coefficient of impact, which encompasses all aspects of the problem and the effects of different parameters, needs lots of research. The solution to various bridges in terms of geometry, the position of the supports and others with respect to dynamic loads is not the same; at the same time, the dynamic loads, depending on the type of load, the speed of transport, the number of loading lines, and the possibility of concurrency, will cause different effects on the structure.

Usually, in the research performed for equivalency of the dynamic load caused by the movement of the train in static form, the relations are only a function of the length of the span of the bridge, and other factors such as the speed of the train, weight of the train and the distance between the axes are not considered [5]. While employing the method of equating dynamic loads to static loads simplifies the process and reduces the time required to obtain results, it does not take into account all the factors influencing the load generated by moving transport. As a result, the outcomes are approximate and may differ slightly from actual conditions. Also, usually, the analyses done by different regulations yield different results. In this research, considering parameters like velocity, weight of the train, the distances of the axes of the train, and the damping of the environment, the dynamic load caused by the movement of the train has been modeled using two methods: analytical and computer modeling. In different conditions for the train, the vertical displacement over the ceiling of the Tehran metro line 3 tunnel has been obtained.

The Finite Difference Method (FDM) is a prominent numerical approach utilized for approximating solutions to partial differential equations. In numerical analysis, FDM is a class of numerical techniques for solving differential equations by approximating derivatives with finite differences. Both the spatial domain and time domain (if applicable) are discretized, or broken into a finite number of intervals, and the values of the solution at the endpoints of the intervals are approximated by solving algebraic equations containing finite differences and values from nearby points [6]. This technique involves the discretization of a continuous domain into a grid, where derivatives are estimated through difference equations. FDM is adaptable and applicable to a range of equations, including those related to fluid dynamics and the propagation of electromagnetic waves. The subsequent sections will discuss the fundamental elements of FDM.

The essential components of FDM include Discretization, Difference Schemes, and Error Analysis. The continuous domain is segmented into a grid, with each grid point corresponding to a discrete value of the function under consideration [7]. Various difference schemes, such as forward, backward, and central differences, are employed to approximate the derivatives. For example, the Crank-Nicolson method is particularly effective for addressing time-dependent problems [8].

FDM facilitates the computation of error norms, which are crucial for evaluating the accuracy of numerical solutions in comparison to known exact solutions.

The literature highlights several applications of FDM. It is employed to model intricate fluid behaviors, such as in Darcy–Brinkman flow, effectively capturing the influence of porous media on fluid dynamics [9]. Additionally, the Finite Difference Time Domain (FDTD) method represents a specific application of FDM for simulating electromagnetic waves, recognized for its simplicity and efficacy [10]. Despite its strengths as a numerical simulation tool, FDM may encounter challenges related to stability and convergence, particularly in complex geometrical configurations or highly nonlinear scenarios. In such cases, alternative methods, such as finite element methods, may offer more reliable solutions [11].

The execution of a numerical simulation aimed at evaluating the impact of train load dynamics on the stability of tunnels encompasses several critical phases, predominantly employing the Finite Difference Method (FDM) alongside various numerical strategies. This methodology effectively integrates the interplay between train dynamics, tunnel architecture, and the geological environment. The model formulation can be summarized as follows:

- Coupled Dynamics Framework: A three-dimensional (3D) time-domain framework is constructed, which includes the interactions among the train, track, tunnel, and soil. The tunnel is represented as a cylindrical dual-layer thin shell, supported by a viscoelastic layer to reflect soil influences [12].
- Transformation of Equations: The governing partial differential equations are reformulated into ordinary differential equations, thereby enabling the utilization of numerical techniques such as FDM [12].
- Dynamic Load Integration: The simulation accounts for the vibrational effects produced by moving trains, focusing on the mechanical vibrations arising from the rail-sleeper assembly [13].
- Boundary Condition Specification: The precision of boundary conditions and the quality of the mesh are vital for accurately capturing pressure wave propagation and structural responses [14].
- Evaluation of Structural Responses: The outcomes of the simulation are scrutinized to evaluate the stability of tunnel linings under diverse loading scenarios, including the characteristics of cracks and the properties of the surrounding rock [15].
- Assessment of Long-term Behavior: The model also addresses the long-term implications of cyclic loading on tunnel structures and adjacent geological formations [12]. Although numerical simulations yield significant insights into tunnel stability under dynamic loading conditions, they may not encompass all real-world complexities, such as unforeseen geological changes or extreme loading events. Therefore, field testing and ongoing monitoring are crucial for thorough evaluations.

2. Review of Train Load Evaluation Literature

Evaluating railways in response to train loads is essential for maintaining the safety and functionality of these structures. Numerical simulation has become an invaluable method for examining the dynamic behavior of railways subjected to different train loads. This section offers an in-depth exploration of the significance of numerical simulation in this evaluation, emphasizing important methodologies, applications, and findings from recent studies. Numerical modeling is a prevalent technique utilized to assess the dynamic response of railway bridges under train loads. This method entails breaking down the bridge structure into a mesh of elements and solving the equations of motion to ascertain displacements, stresses, and other dynamic responses.

One of the primary benefits of numerical simulation lies in its capacity to model the intricate interactions among the train, track, and bridge. For example, a researcher has developed a finite element model specifically for a viaduct, which integrates the bridge-track-train system. This model treats the viaduct platform, track elements, and train as rigid bodies, incorporating discrete springs and dampers to represent the suspension systems. Such a methodology facilitates the identification of resonant velocities and enables the examination of time histories related to displacements and stresses [16].

Additionally, other studies investigate the fatigue behavior of steel bridge components through multi-scale dynamic analysis and fracture mechanics. These investigations take into account the influence of track irregularities and train speeds on the propagation of fatigue cracks. The findings underscore the significance of precise dynamic analysis in assessing fatigue and informing maintenance strategies [17].

Parametric studies play a crucial role in elucidating the impact of various factors on the dynamic performance of railway bridges. The subsequent investigation presents a parametric analysis focused on the dynamic response of railway bridges subjected to heavy-haul trains. This analysis takes into account several parameters, including the length of the bridge span, the speed of the train, and the mass ratio. The findings underscore the significance of incorporating inertial effects, especially when the mass ratio between the train and the bridge surpasses 40 percent [18].

Additionally, other researchers evaluate the reliability of railway bridges in the context of high-speed traffic, factoring in track quality and system variability. This study employs a stochastic train-bridge model alongside Hypercube sampling to assess the influence of random variables and rail irregularities on structural reliability. The outcomes highlight the necessity of considering uncertainties when evaluating the integrity of railway bridges [19].

In a similar vein, another study emphasizes the significance of updating numerical models through the use of ambient vibration test data. The findings indicate that these revised numerical models can effectively represent the dynamic behavior of high-speed railway bridges, with discrepancies in vertical and torsional modes being less than 5% when compared to measured data. This methodology proves particularly advantageous for evaluating dynamic stability, eliminating the necessity for costly field load tests [20].

Additionally, other researchers investigate the impact of coupling beam modeling on structural accelerations during the passage of high-speed trains. Their research reveals that accounting for the dynamic interactions among the train, track, and bridge results in more precise predictions of structural vibrations. The coupling beam model, which conceptualizes the track and its supporting structure as two vertically interconnected beams, can diminish structural accelerations by as much as 80% in comparison to basic moving load models [21].

Research has similarly introduced a numerical simulation framework for updating and validating truss railway bridges. This study combines operational modal analysis with a sensitivity-based model updating technique to improve accuracy. The findings reveal a notable decrease in frequency errors, underscoring the effectiveness of this approach for vibration-based structural health monitoring. The precision of numerical models is essential for the dependable evaluation of railway bridges. This research highlights the significance of model updating and validation, especially in the context of vibration-based assessments. The study illustrates that employing sensitivity-based model updating can lower average frequency errors from 11% to 3%, resulting in more precise predictions of dynamic responses [22].

Furthermore, machine learning methodologies have been investigated for forecasting the dynamic responses of railway bridges. The research presents a deep Long Short-Term Memory network designed to predict bridge responses when subjected to moving trains. This framework integrates the train-bridge coupling mechanism with deep learning algorithms, taking into account the duration of train loads. The results demonstrate that this network can effectively predict bridge displacements and accelerations, achieving greater accuracy in displacement predictions. The application of machine learning can significantly alleviate computational demands while preserving acceptable accuracy levels [23].

The interaction among the train, track, and bridge is a vital component of railway bridge dynamics. A study examines the dynamic performance of three slab ballastless tracks under various high-speed train loads [24].

Table 1 presents a succinct comparison of different numerical modeling techniques, emphasizing their essential characteristics and applications in evaluating railway structures under train loads.

Table 1. A concise comparison of various approaches for evaluating dynamic train loads.

Modeling Approach	Key Features	Application	Researchers
Finite Element Modeling	Discretization of structure into mesh elements; solves equations of motion	Analysis of displacements, stresses, and dynamic responses under train loads	Szurgott & Kozera[16]
Machine Learning	Predicts dynamic responses using deep learning algorithms	Real-time monitoring and structural health assessment	Vataev et al.[13]
Coupling Beam Modeling	Represents track and supporting structure as coupled beams	Accurate prediction of structural vibrations and accelerations	Gorbacheva & Poleviko[11]
Semi-Analytical Approach	Combines modal expansion and component mode synthesis	Dynamic response analysis considering soil-structure interaction	Konig et al.[25]
3D Multibody Dynamics	Treats train as a 3D multibody assembly with kinematic constraints	Dynamic interaction between train and steel-truss arch bridge	Zeng et al.[26]
Ambient Vibration Test	Updates numerical models using measured data	Assessment of dynamic stability without field load tests	Kim et al.[20]
Stochastic Modeling	Considers track quality and system randomness	Reliability assessment under high-speed traffic	Yuan[19]
Multi-Scale Dynamic Analysis	Combines vehicle-bridge analysis with fracture mechanics	Fatigue evaluation of steel bridge details	Li & Wu[17]

Numerical simulation plays a vital role in evaluating railway bridges under the influence of train loads. By employing numerical modeling and conducting parametric analyses, engineers can gain valuable understanding of the dynamic behavior of these structures. The integration of advanced modeling techniques with empirical data and validation procedures improves the accuracy and reliability of numerical models. As the infrastructure for high-speed and heavy-haul railways expands, the development of resilient numerical tools will continue to be a key area of emphasis in both research and practical implementation.

3. Establishing the Problem Framework and Presenting the Case Study

3.1. Tehran Subway Line 3 Tunnel

For the purpose of expanding the southern part of Tehran metro line 3, a part of the tunnel with the Tehran-Ahvaz railroad lines is intersected. Therefore, for the crossing of railroad trains, there is a need to build a bridge over the metro tunnel as the ceiling of this underground space. The shape of the cross-section of this tunnel is rectangular and is dug 2 meters from the surface of the earth, with a width of 9 meters and a height of 7 meter. In the initial design for the construction of this part of the tunnel, the method of cut and cover has been used, and to the depth under consideration, digging and trenching services are performed, and the systems of support, which contain piles and shotcrete, are installed. Then, a concrete beam that has been made beforehand is placed on the ceiling of the tunnel and with the use of concrete and completion of the other parts of the ceiling, railroad lines are built over the ceiling of the tunnel.

In **Tables 2–6**, the geotechnical specifications of the soil, piles, shotcrete, concrete and precast concrete beam used in this research are presented. Also, due to the proximity of the tunnel to the surface of the earth, underground water is at a large distance from the tunnel and does not affect the analysis of this design.

Table 2. Properties of soil layer of Tunnel of Tehran Subway [27].

Parameter	Symbol	Unit	Value
Specific weight	γ	Kg m^{-3}	2000
Modulus of elasticity	E	MPa	50
Poisson's ratio	v	-	0.4
Friction angle	φ	°	37
Cohesion	C	KPa	11.76
Ratio of horizontal stress to vertical stress	K	-	0.74

Table 3. Properties of piles in Tunnel of Tehran Subway [28].

Parameter	Symbol	Unit	Value
Specific weight	γ	Kg m^{-3}	2550
Modulus of elasticity	E	GPa	21
Poisson's ratio	v	-	0.15
Nnormal coupling spring stiffness per unit length	K_n	N m^{-1}	1×10^{10}
Normal coupling spring cohesion per unit length	C_n	N m^{-1}	1×10^{8}
Normal coupling spring friction angle	ϕ_n	°	0
Shear coupling spring stiffness per unit length	K_s	N m^{-1}	1×10^{10}
Shear coupling spring cohesion per unit length	C_n	N m^{-1}	1×10^{8}
Shear coupling spring friction angle	ϕ_n	°	0

Table 4. Properties of shotcrete for tunnel support in Tunnel of Tehran Subway [29].

Parameter	Symbol	Unit	Value
Specific weight	γ	Kg m^{-3}	2550
Modulus of elasticity	E	GPa	25
Poisson's ratio	v	-	0.2

Table 5. Properties of concrete for tunnel support in Tunnel of Tehran Subway [30].

Parameter	Symbol	Unit	Value
Specific weight	γ	Kg m^{-3}	2400
Modulus of elasticity	E	GPa	26.8
Poisson's ratio	v	-	0.2

Table 6. Properties of reinforced concrete slab for tunnel support in Tunnel of Tehran Subway [31].

Parameter	Symbol	Unit	Value
Specific weight	γ	Kg m^{-3}	2500
Modulus of elasticity	E	GPa	29.4
Poisson's ratio	v	-	0.2

3.2. The Dynamic Analysis of the Bridges under the Movement of a Single Load Transport

The simplest method for the dynamic analysis of the bridges is the use of a moving load over the bridge. In this method, a load according to **Figure 1** with the speed of v is applied to a simple beam with the length of L, an elasticity modules of E, and a moment of inertia of I. By using Equation (1), the displacement of any point from the beam (x) during the time t can be obtained. By multiplying the results obtained from this relation by the number of loads under consideration for crossing over the beam, the displacement after the crossing of the vehicle is determined [32]. In Equation (1), ω_n is the frequency of vibration, and its value is obtained by Equation (2). M in Equation (2) is the mass per unit length of the beam. The value of the coefficient α is also obtained from Equation (3).

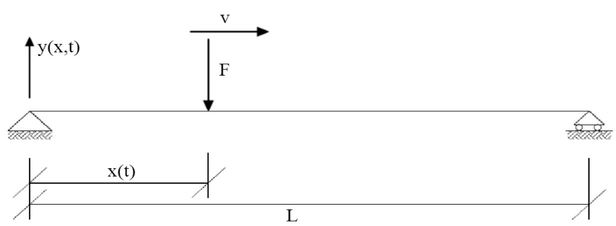

Figure 1. Dynamic analysis of simple beam under a moving load [32].

$$y(x,t) = \frac{96\,FL^3}{48\,E\pi^4}\sum_{n=1}^{\infty}\frac{1}{n^4\left(1-\alpha^2/n^2\right)}\left(\sin\left(\frac{n\pi v}{L}t\right) - \frac{\alpha}{n}\sin\left(\omega_n t\right)\right)\sin\left(\frac{n\pi x}{L}\right) \quad (1)$$

$$\omega_n = \left(\frac{n\pi}{L}\right)^2\sqrt{\frac{EI}{M}} \quad (2)$$

$$\alpha = \frac{\pi v}{\omega_1 L} \quad (3)$$

3.3. The Principles and Foundations of the Dynamic Behavior of the Bridges under Moving Concentrated Loads

Another method of evaluating the dynamic solution of bridges under the movement of a vehicle is to consider the bridge as a simple beam and its dynamic analysis under the movement of concentrated loads. This method, in addition to precision, is relatively simple and makes the possibility of lots of analysis in a short time. For the evaluation of the theory governing the movement of the train on the bridge with a simple span, according to **Figure 2**, a beam with a span of length L and a fixed cross-section is considered. The train is also considered as a series of concentrated loads with the same distance that moves with a fixed speed of v over the beam. The distance between these loads is d, and the magnitude of each load is p.

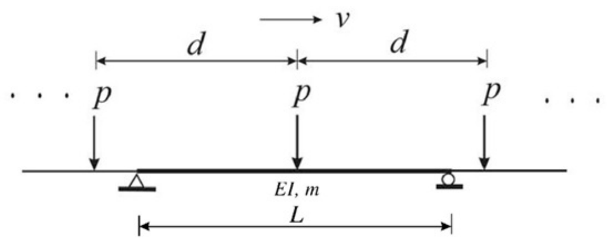

Figure 2. A view of the model of the train moving over the bridge as moving concentrated loads [33].

Equation (4) is the equation of the dynamic movement of the moving concentrated loads over this beam. In this relation, u' is the derivative of u relative to the x coordinate and $\dot{u}$ is the derivative of u relative to time.

$$m\ddot{u} + cI\dot{u}'''' + EIu'''' = P\sum_{j=1}^{N} \delta\left(x - v(t-t_j)\right).\left[H(t-t_j) - H\left(t - t_j - \frac{L}{v}\right)\right] \tag{4}$$

The rest of the signs are as follows.

M: The mass per unit length,

C: Damping matrix,

E: Modulus of elasticity,

$t_j = (j-1)\, d/v$: The time of arrival of the j'th load,

$u(x,t)$: Vertical displacement of the beam,

t: The time of crossing total loads,

I: Moment of inertia,

N: The number of moving loads.

δ: is the Dirac delta function which is expressed as Equation (5).

$$\delta(x) = \begin{cases} \infty & x = 0 \\ 0 & x \neq 0 \end{cases}; \qquad \int_{-\infty}^{\infty} \delta(x)dx = 1 \tag{5}$$

$H(t)$: is the unit step function which is defined as Equation (6).

$$H(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases} \tag{6}$$

Equations (7) and (8) show the boundary conditions of Equation (4).

$$u(0,t) = u(L,t) = 0 \tag{7}$$

$$EIu''(0,t) = EIu''(L,t) = 0 \tag{8}$$

With the assumption that the bridge is at rest, the initial conditions are also determined by Equation (9).

$$u(x,0) = \dot{u}(x,0) = 0 \tag{9}$$

The vertical displacement of the bridge is expressed by Equation (10).

$$u(x,t) = \sum_{n=1}^{\infty} q_n(t).\sin\left(\frac{n\pi x}{L}\right) \tag{10}$$

In Equation (10), q_n is the generalized coordinate of the n'th mode, and the function of its shape is $sin(n\pi x/L)$. By replacing this relation in place of u in the differential equation of the vibration of the bridge and multiplying both sides of the relation by $sin(n\pi x/L)$ and integrating over the length of the bridge relation 11 is obtained. In this relation, the values of ω_n and $F_n(t)$ are calculated from Equations (2) and (12) respectively. Also, ξ is the damping of the bridge.

$$\ddot{q}_n + 2\xi\omega_n\dot{q}_n + \omega_n^2 q_n = F_n(t) \tag{11}$$

$$F_n(t) = \frac{2P}{mL}\sum_{j=1}^{N}\left[\sin\left(\frac{n\pi v(t-t_j)}{L}\right).H(t-t_j) + (-1)^{n+1}\sin\left(\frac{n\pi v\left(t-t_j-\frac{L}{v}\right)}{L}\right).H\left(t-t_j-\frac{L}{v}\right)\right] \tag{12}$$

Equations (13) and (14) are the solutions of the differential Equation (11) using the method of Duhamel.

$$q_n(t) = \frac{1}{m\omega_{dn}} \int_0^t F_n(\tau).e^{-\xi\omega_n(t-\tau)} \sin\left(\omega_{dn}(t-\tau)\right) d\tau \quad (13)$$

$$q_n(t) = \frac{2PL^3}{n^4 EI\pi^4} \sum_{j=1}^{N} \frac{1}{(1-S_n^2)^2 + 4(\xi S_n)^2}. \left[A.H(t-t_j) + (-1)^{n+1} B.H\left(t - t_j - \frac{L}{v}\right)\right] \quad (14)$$

In these equations, S_n, A and B are calculated from Equations (15) – (17).

$$S_n = \frac{n\pi v}{\omega_n L} \quad (15)$$

$$A = (1-S_n^2)\sin\frac{n\pi v}{L}(t-t_j) - 2\xi S_n.\cos\left(\frac{n\pi v}{L}\right)(t-t_j) + e^{-\xi\omega_n(t-t_j)}.\left[2\xi S_n \cos\omega_{dn}(t-t_j) + \frac{S_n}{\sqrt{1-\xi^2}}(2\xi^2 + S_n^2 - 1)\sin\omega_{dn}(t-t_j)\right] \quad (16)$$

$$B = (1-S_n^2)\sin\frac{n\pi v}{L}\left(t-t_j-\frac{L}{v}\right) - 2\xi S_n.\cos\left(\frac{n\pi v}{L}\right)\left(t-t_j-\frac{L}{v}\right) + e^{-\xi\omega_n\left(t-t_j-\frac{L}{v}\right)}.\left[2\xi S_n \cos\omega_{dn}\left(t-t_j-\frac{L}{v}\right) + \frac{S_n}{\sqrt{1-\xi^2}}(2\xi^2 + S_n^2 - 1)\sin\omega_{dn}\left(t-t_j-\frac{L}{v}\right)\right] \quad (17)$$

In Equations (16) and (17), the parameter ω_{dn} is the damped frequency of vibration of the n'th mode of the beam, and its value is obtained from Equation (18).

$$\omega_{dn} = \omega_n\sqrt{1-\xi_n^2} \quad (18)$$

By calculating $q_n(t)$ from Equation (14) and substituting it into Equation (10), the displacement of any point of the beam in the distance $x = 0$ to $x = L$ and in the time t is calculable [33].

3.4. Simulation of Dynamic Load of the Train

In order to accurately simulate the behavior of a train under dynamic loads, it is essential to consider various factors that contribute to the overall dynamic load function of the model. These factors include the distance of the axis of the wheels, the force exerted by each axis of the train (which is calculated based on the weight of each wagon), and the speed at which the train is traveling. To determine the dynamic load function, the dimensions of the train's wagons, their direction of movement, and the distances between the axes of each wheel must be carefully analyzed according to a specified reference point, as shown in **Figure 3.** By taking into account the velocity of the train and the weight of each individual wagon, a detailed diagram illustrating the force applied by the wheels of the wagons over time at a specific point on the railroad can be created. This comprehensive analysis and modeling of the dynamic load function are crucial for understanding the structural integrity and performance of the train under varying operating conditions. By accurately determining the forces at play, engineers can make informed decisions to ensure the safety and efficiency of train operations.

Consider a scenario where we have a train consisting of multiple wagons, each equipped with wheels that exert a force F on the railroad track. Let's focus on the first wagon of the train and analyze the application of forces over time. At the start of our analysis, we designate t = 0 as the origin of the assumed imaginary axis. In this setup, the force F exerted by the first wheel of the first wagon is applied to the railroad. It is crucial to note that the time interval between the forces applied by the two adjacent wheels is relatively small, denoted as $t_1 = x/v$, where x is the distance between the wheels and v represents the velocity of the train.

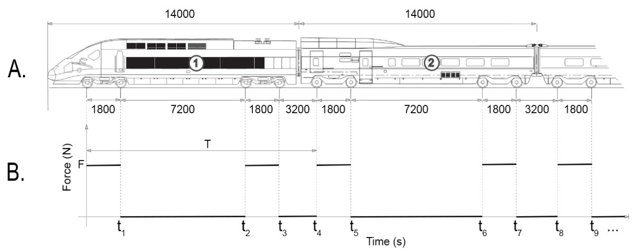

Figure 3. Dimensions of the wagons of the train, and the force applied by the wagon wheels relative to time (dimensions in terms of millimeters).

Due to the short time frame, we can approximate these forces as a fixed continuous linear force with a magnitude of F. This fixed linear force is then applied within the time interval of t_1. As time progresses, once t_1 elapses, the force applied by the initial wheels ceases. It is not until the moment t_2, which marks the instance when the forces of the two other adjacent wheels of the first wagon come into play, that the applied force drops to zero. Similar to the initial scenario, t_2 signifies the initiation of the forces exerted by these two wheels, with the time interval of force application extending from t_2 to t_3. Following the timeline, after the force application period from t_2 to t_3 concludes, the wheels of the first wagon come to a temporary stop in their force application until time t_4, where no force is exerted. This sequence of force application and cessation offers a detailed insight into how the forces generated by the wheels of the wagon interact with the railroad track over specific time intervals. This, in turn, plays a crucial role in determining the dynamics and movement of the train. Moreover, time t_4 marks the completion of one full cycle of force application by the wagon's wheels. Subsequently, this cycle of applying force repeats itself, as illustrated in Figure 3. The continuous cycle of force application and cessation over time is crucial in understanding the overall behavior of the train's movement. In summary, the relationship between the applied force and time during the time frame from zero to t_4 is captured by Equation (19). This detailed analysis of force dynamics provides valuable insights into the intricate interplay between the forces exerted by the train's wheels and their impact on the track, ultimately influencing the train's motion and trajectory [34].

$$f(t) = \begin{cases} F\,, & 0 \le t < t_1 \\ 0\,, & t_1 \le t < t_2 \\ F\,, & t_2 \le t < t_3 \\ 0\,, & t_3 \le t < t_4 \end{cases} \tag{19}$$

The function in Equation (19), presented as a periodic function with a periodicity of $T = t_4$, exhibits a regular pattern that repeats every t_4 units of time. This type of function can be effectively analyzed using Fourier series, a mathematical tool that allows us to represent periodic functions as a sum of sinusoidal and cosine functions. In Equation (20), the function f(t) can be expressed as a continuous function using the Fourier series, with coefficients a_0, an, and b_n determining the amplitudes of the individual sinusoidal and cosine terms. These coefficients are calculated through specific formulas outlined in Equations (21)–(23), enabling us to decompose the periodic function into its constituent components and analyze its behavior over time. By utilizing Fourier series, we can gain valuable insights into the structure and properties of periodic functions, facilitating in-depth analysis and interpretation of their behavior.

$$F(t) = a_0 + \sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{2n\pi t}{T}\right) + b_n \sin\left(\frac{2n\pi t}{T}\right)\right] \tag{20}$$

$$a_0 = \frac{1}{T}\int_0^{t_4} f(t)\,dt \tag{21}$$

$$a_n = \frac{2}{T}\int_0^{t_4} f(t)\cos\left(\frac{2n\pi t}{T}\right)dt \tag{22}$$

$$b_n = \frac{2}{T}\int_0^{t_4} f(t)\sin\left(\frac{2n\pi t}{T}\right)dt \tag{23}$$

In the year 2014, Jiang and his team conducted a detailed experimental test aimed at replicating and analyzing the motion of a train in order to assess the dynamic loads experienced by the railroad as a result. The study involved the use of hydraulic jacks, controlled by a computer system, which were strategically positioned along the railroad at equal intervals based on the locations of the train wheels. By adhering to the principles explained in previous studies, the hydraulic jacks applied loads equivalent to the axial load of the train wheels at regular intervals during the experiment. The results obtained from this comprehensive test demonstrated a high level of correlation with real-world field measurements, indicating the validity and accuracy of the methodology employed [35].

Figure 4 illustrates the dynamic loading process carried out in this experiment, providing a visual representation of the precise procedures followed by the researchers.

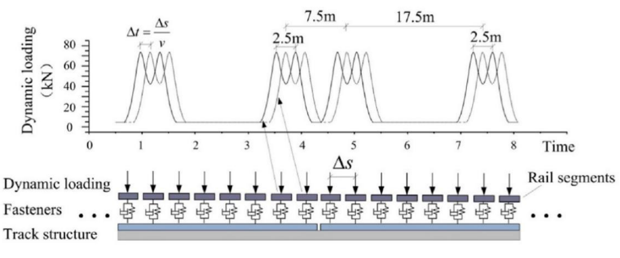

Figure 4. Dynamic loadings in the sequential loading system to simulate train moving loads at the track structure [36].

In this particular study, the researchers focused solely on analyzing the dynamic load exerted on the precast concrete beam positioned on the ceiling of the tunnel. They intentionally chose to omit the application of the load in the sections preceding and following the beam in order to streamline their calculations. This decision was made because factoring in these additional areas would significantly prolong the time required for computations within the software, rendering it practically impossible to obtain a meaningful result. Moreover, it was determined that the impact of the train passing through this specific region of the tunnel is minimal and therefore negligible. As a result, the researchers confidently disregarded the effects of the train's passage in this area, as they had determined it to be inconsequential to their overall analysis.

As mentioned previously, the dynamic load calculated through this simulation method is applicable to a single point within the model, making it most suitable for addressing two-dimensional problems. In the context of the current research focusing on examining three-dimensional models, a modified approach is taken. The length of the area where the train's load will be applied is divided into equal segments. By considering the time at which the first wheel of the train enters each segment, the dynamic load is exerted on them accordingly.

The calculation of the time at which the dynamic load is applied involves the length of each segment and the velocity of the train, following the formula $t = x/v$. This approach ensures that the dynamic load is accurately distributed to each segment as the train moves through them. Once the last wheel of the train passes over the last segment, the dynamic load in all segments is removed, completing the simulation.

Implementing this method allows for a reasonably accurate portrayal of the train's motion in a three-dimensional scenario. This detailed and systematic approach enhances the precision and reliability of the simulation results, contributing valuable insights to the research study.

In **Figure 5**, the diagram illustrates the method of partitioning a given system into distinct parts, each with different sizes and functionalities. This visual representation allows viewers to clearly see how the system is divided into specific components, with each part serving a unique purpose or carrying out a specific function. This detailed breakdown provides valuable insight into the organization and structure of the system, aiding in a comprehensive understanding of its inner workings and relationships between different components.

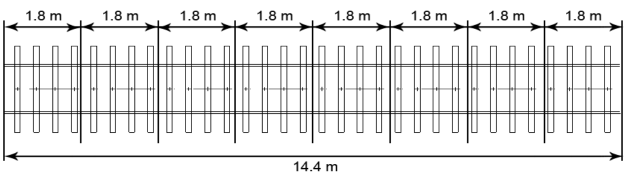

Figure 5. The method of partitioning the areas considered for the passing of the train and the size of each part.

Through meticulous modelling and multiple iterations of adjusting distances, it was discovered that the length of various parts had no significant impact on the results. The experiment focused on the rate of displacement in the center of a bridge span as a train traveling at 80 kilometers per hour passed by. Two different scenarios were tested: one with 8 parts measuring 180 centimeters each, and another with 16 parts measuring 90 centimeters each. Interestingly, the results from these two cases showed a minuscule difference of only 0.05 millimeters. In **Figure 6**, the graph visually represents this slight variance, highlighting the negligible effect of the length of the parts on the overall outcome.

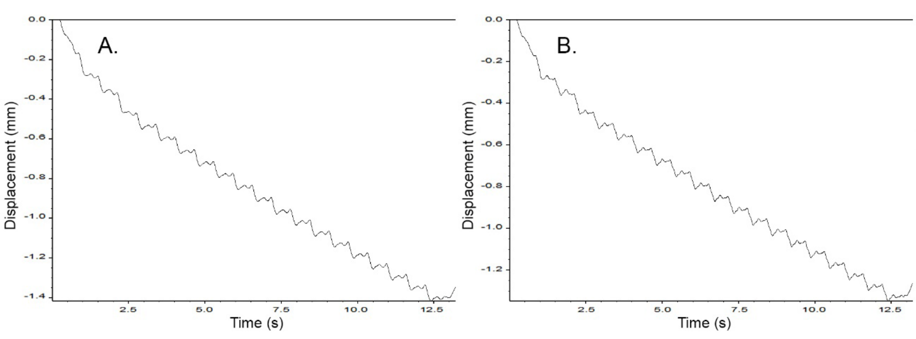

Figure 6. The displacement obtained in the two cases: (a) 16 parts each of 90 centimeters, and (b) 8 parts each of 180 centimeters.

4. Results and Discussion

After modelling in the Finite Difference Method, the results obtained from the dynamic loading on the model in different conditions of the motion of the train have been examined. For this purpose, the parameters of the velocity of the train, weight of the wagon, number of wagons and the rate of damping of the environment have been analyzed in several different amounts, untill the effect of these parameters on the rate of displacement of the middle of the span of the bridge is determined. Then, the results obtained from the analysis of the model in the software are compared to the results obtained from the methods of dynamic analysis of the bridge under the motion of a single load moving and concentrated loads moving. These comparisons have been done under the same conditions for each of the three methods. In these examinations, the distances of the axis of the wheels of the train have been considered according to **Figure 3**.

Initially, a train with 20 wagons, each weighing 100 tons, and damping of 1% for the environment, with five velocities of 40, 60, 80, 100 and 120 kilometers per hour have been considered. **Figure 7** shows the results of these three analyses for the different velocities of the train. Considering this figure, it can be concluded that with the increase in the velocity of the train, the rate of vertical displacement in the middle of the span of the bridge decreases. The reason for this event is the shorter time of loading over the model at higher speeds.

Figure 8 shows the results of examinations of three different trains with weights of 100, 110 and 120 tons for each wagon. In this examination, the velocity of the trains has been considered to be 40 kilometers per hour, and the number of wagons is 20, with the damping of the environment assumed to be 1%. The results show that with the increase in the weight of the wagons, the change in shape over the passage will also increase.

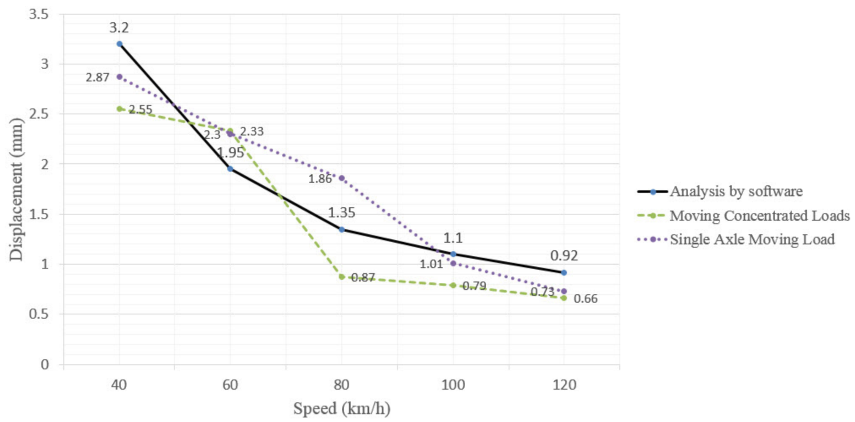

Figure 7. Comparison of the three methods of analysis at different speeds.

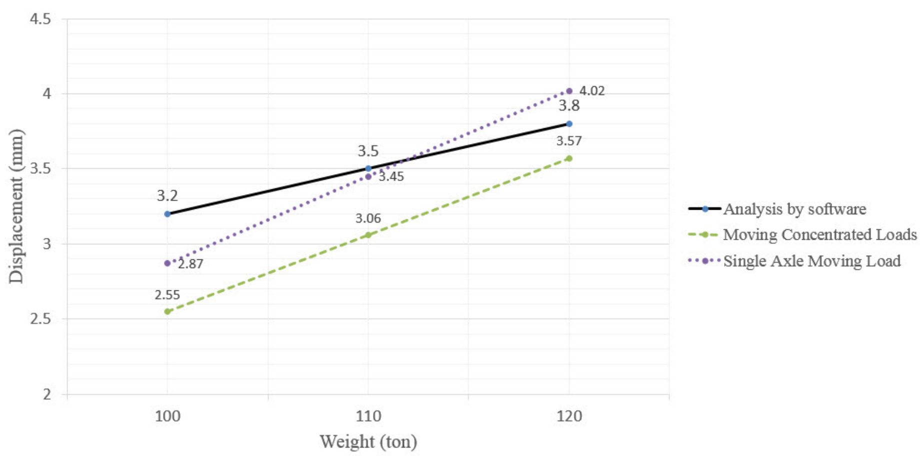

Figure 8. Comparison of the three methods of analysis for different weights of the wagon.

Since usually in dynamic modelling, the damping of the model is chosen between 1% to 3%, in this examination, the damping of the model in three states of 1%, 2%, and 3% and a number of 10 wagons, each weighing 120 tons, with a velocity of 40 kilometers per hour have been examined. The results of this examination are shown in **Figure 9**. From **Figure 9**, it can be concluded that the damping of the environment has an insignificant effect on the change in shape of the model, and with the increase in the damping, the vertical displacement in the middle of the span of the bridge increases by a small amount.

Usually freight trains have a maximum of 35 wagons. Therefore, for examination of the effect of the number of wagons on the rate of vertical displacement of the bridge, the train has been modelled in four cases of 20, 25, 30 and 35 wagons. With respect to the highest rate of vertical displacement obtained in the previous sections, the velocity of the train is considered to be 40 kilometers per hour, the weight of the wagons is 120 tons, and the damping of the environment is 1 percent. **Figure 10** shows the results obtained from this analysis.

Figure 10 shows that the number of wagons has a large effect on the rate of vertical displacement of the middle of the span of the bridge. Due to the increase in the time interval and the number of cycles of loading with the increase in the number of wagons, the rate of displacement over the bridge will also increase.

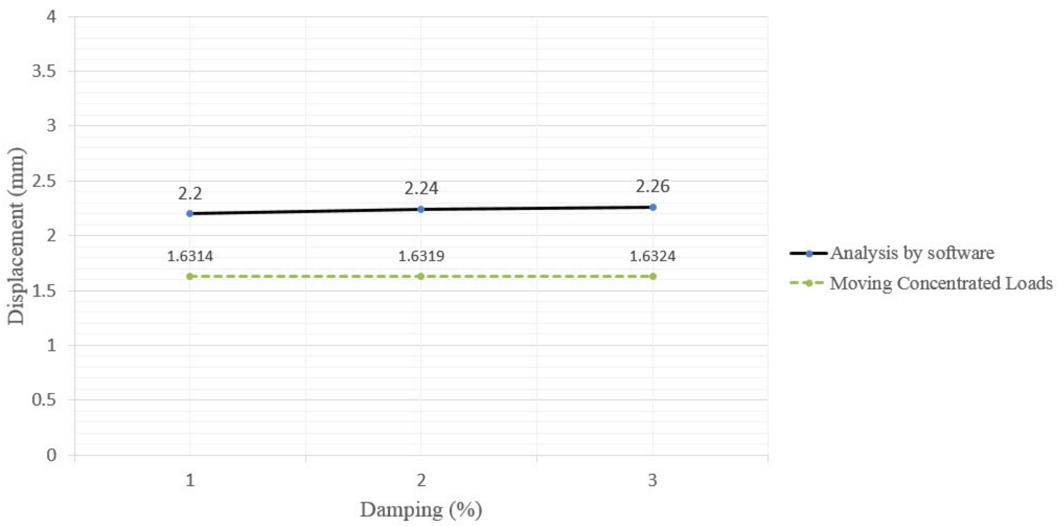

Figure 9. Comparison of two methods of analysis in different methods.

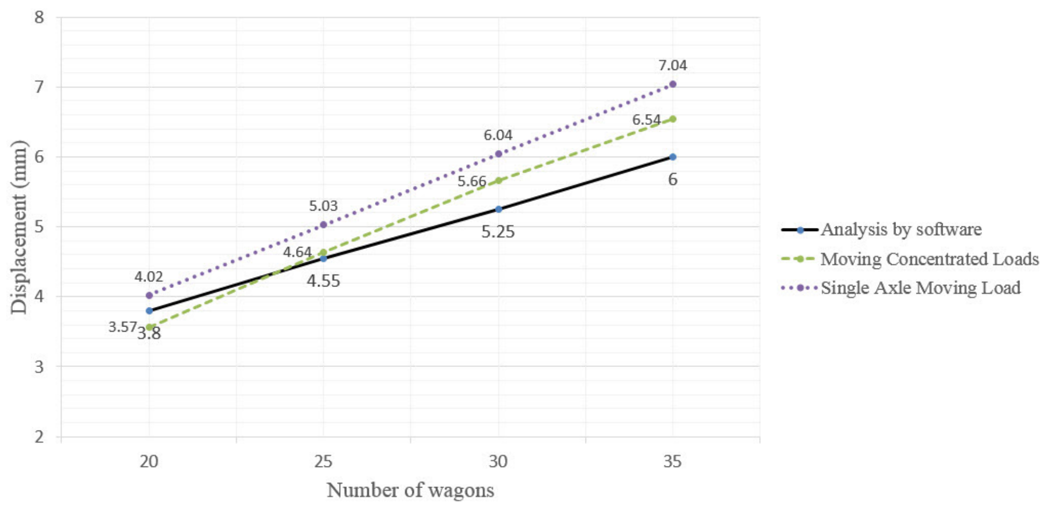

Figure 10. Comparison of three methods of analysis for different numbers of wagons.

Modeling indicated that both the weight and number of wagons significantly affect the vertical displacement at the midpoint of the bridge span compared to other parameters. As the number of wagons increases, the loading cycles and the duration of loading also rise, leading to greater displacement on the bridge and a subsequent reduction in the design safety factor. For a train consisting of 35 wagons, the design safety factor was calculated to be 2.34, suggesting that the design stability remains in a favorable condition.

Examination of the diagrams obtained from the results of the different analyses shows good compatibility with each other from which the validity of the results obtained from the analysis of the model.

Overall, it can be observed that current regulations do not adequately address significant factors, such as the impact of successive loads and the potential for intensification, when examining the dynamic effects of rail loads on bridges. Recent studies have highlighted the shortcomings of the dynamic impact coefficient relationships outlined in design regulations. Findings indicate that various factors (including the type of bridge, span length, train speed, axle spacing, train weight, and support conditions) play a crucial role in the dynamic responses of bridges. In fact, the research on rail load effects on railway bridges parallels that of highway bridges, despite the differences in rail load types and their application sequences compared to highway traffic loads. Consequently, the impact coefficient relationships proposed in AASHTO and Eurocode regulations for railway bridges are not precise, underscoring the need for dynamic analyses to accurately assess the effects of rail loads.

The approach proposed in this research offers numerous advantages; however, its application may also involve specific limitations. This section delineates the constraints associated with the model.

The model necessitates a structured grid, which can pose challenges when dealing with complex geometries. Irregular domains may necessitate intricate grid generation processes or result in inaccuracies. The precision of the results is significantly influenced by the grid spacing. A finer grid enhances accuracy but also escalates computational expenses. Moreover, the method may face convergence difficulties for certain problem types, especially those characterized by discontinuities or steep gradients. Establishing boundary conditions can prove to be problematic, particularly for intricate or unconventional boundaries. Misapplication of these conditions can result in considerable errors in the final solution. The selection of time step and grid spacing must adhere to specific stability criteria, which may limit the permissible time step size in time-dependent scenarios. The proposed model is typically more appropriate for problems defined within simple geometries, such as rectangular grids. Additionally, errors in the numerical solution can propagate throughout the grid, leading to cumulative inaccuracies, particularly in long-duration simulations. While this method can be applied to nonlinear problems, managing nonlinearity often necessitates iterative techniques that can complicate implementation and heighten computational costs. In comparison to the Finite Element Method (FEM) or the Finite Volume Method, FDM is less adaptable in addressing varying material properties and complex boundary conditions. Furthermore, the model's output may require substantial post-processing to derive meaningful insights.

The continued development of proposed simulation techniques is essential for improving the assessment of railway bridges under train loads. Future research should focus on integrating advanced machine learning algorithms with traditional models to enhance prediction accuracy and computational efficiency. Additionally, the consideration of environmental and operational factors, such as temperature effects and track irregularities, should be further explored to develop more comprehensive models.

5. Conclusions

In this research, by simulating the motion of the train, the effect of the dynamic load created from this movement over part of the Tehran metro line 3 tunnel which intersects the trans-country railroad of Tehran-Ahvaz, has been examined. The results obtained from the computer modelling have been compared with two methods of analysis till the vertical displacement rate in this part of the ceiling of the tunnel of Tehran metro line 3 in different conditions is determined. With respect to the subject discussed and the analysis done, the results obtained from these analyses can be expressed as follows. With an increase in the velocity of the vehicle, due to the decrease in the time interval of loading over the bridge, the maximum displacement of the middle of the span of the bridge decreases. An increase in the weight of the wagons of the train increases the rate of displacement of the middle of the span of the bridge. With an increase in the number of wagons passing over the bridge, the number of cycles of loading and the time interval of loading also increase. As a result, the rate of vertical displacement of the middle of the span of the bridge also increases. Due to low speed of loading relative to the problems related to explosion and earthquake, the amount of damping does not have a significant effect on the rate of vertical displacement of the middle of the span of the bridge. By considering the most critical condition for the trains existing in the railroad lines of Tehran-Ahvaz, the rate of vertical displacement of the middle of the span of the bridge is obtained to be 6 millimeters, and with respect to the regulations of the bridges of the railroad, the factor of safety is 2.34, which confirms the stability of the design under consideration.

Author Contributions

Supervision and administration, F.S.N.; methodology, F.S.N.; data collection, M.N.A.; conceptualization, M.N.A.; conception and design of the analysis, F.S.N.; formal analysis, M.N.A.; data processing, M.N.A.; investigation resources, F.S.N.; data validation, F.S.N.; writing – original draft, M.N.A.; final writing and editing, F.S.N. All authors have read and agreed to the published version of the manuscript.

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The data that support the findings of this study are available on request from the corresponding author.

Conflicts of Interest

The authors declare no conflict of interest. The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript; or in the decision to publish the results.

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